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TRANSONIC AIRFOIL CODES*

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SUMMARY

Three books on supercritical wing sections have been published recently that document and list codes for the design and analysis of transonic airfoils. The codes have had a significant impact on the development of supercritical wing technology. This paper is devoted to several new contributions to the theory on which the codes are based. It has become possible to prescribe the pressure distribution within a reasonable tolerance even over the supersonic portion of a shockless airfoil. The purely subsonic problem of modeling the flow near the trailing edge has been handled in a way that eliminates any appreciable loss of lift in practice. Boundary layer effects are taken into account in an empirically satisfactory fashion. The work has been extended to the case of cascades of airfoils appropriate for the design of compressor and turbine blades. These methods of computational fluid dynamics have produced a family of airfoils that deliver outstanding performance over a wide range of conditions.

INTRODUCTION

In the last few years three books have appeared that list computer codes for the design and analysis of transonic airfoils (refs. 1,2,3). The design code relies on the method of complex characteristics in the hodograph plane to construct shockless airfoils. The analysis code uses artifical viscosity to calculate flows with weak shock waves at off-design conditions. Comparisons with experiments show that an excellent simulation of two-dimensional wind tunnel tests is obtained. The codes have been widely adopted by the aircraft industry as a tool for the development of supercritical wing technology.

SYMBOLS

f   complex analytic function

h   real function

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i	square root of -1
k	real constant
M	Mach number
q	speed
u	function of x and y
u_1	even function of y
u_2	even function of y
x	canonical coordinate
y	canonical coordinate
ξ	characteristic coordinate
η	characteristic coordinate
θ	flow angle
ρ	density
ϕ	velocity potential
ψ	stream function

ANALYSIS AND DESIGN

For analysis, differencing of the partial differential equations of gas dynamics that do not adhere to strict conservation form turns out to give the best representation of boundary layer-shock wave interaction over a broad range of conditions (ref. 2). An improved formula for the wave drag compensates for errors in the conservation of mass across shocks that are of the third order in the shock strength. Modern techniques of conformal mapping and fast Fourier transform have led to an upgraded analysis code that has unusual speed and accuracy (ref. 3).

Corrections for the displacement thickness of the turbulent boundary layer have been made in both the design and the analysis codes. Even more important is an adequate model of the flow near the trailing edge of the airfoil. It suffices to represent the wake by a pair of parallel streamlines across which the pressure balances. Large favorable pressure gradients can be tolerated on the lower surface to provide for heavy aft loading. However, a Stratford distribution should be used at the rear of the upper surface to limit the adverse pressure gradient there so as to avoid separation. Airfoils designed with this in mind perform well over a wide range of conditions.

Shockless airfoils serve as an acceptable mathematical model for the design of supercritical wing sections. Drag creep can be reduced by restricting the size of the supersonic zone in the shockless flow. The design method has been extended to include cascades of airfoils such as occur in compressors and turbines. Present codes can handle gap-to-chord ratios down to unity (see figures 1 and 2). The concept of a supercritical compressor blade has been tested successfully by Harry Stephens of Pratt and Whitney Aircraft in a cascade wind tunnel of the DFVLR in Germany (see figure 3).

The latest version of the design code enables one to assign the pressure distribution with a certain tolerance and still obtain shockless flow when it exists (ref. 3). This is achieved by formulating a new boundary value problem in the unit circle of the complex plane of one of the characteristic coordinates for the gas dynamics equations. It is worthwhile to review briefly the theory underlying the new code, which seems to be quite successful in practice.

In terms of characteristic coordinates ξ and η , the partial differential equations for the velocity potential ϕ and stream function ψ of plane compressible flow can be expressed in the form

$$\phi_\xi = i\,\frac{\sqrt{1-M^2}}{\rho}\,\psi_\xi \quad , \quad \phi_\eta = -i\,\frac{\sqrt{1-M^2}}{\rho}\,\psi_\eta$$

where M is the local Mach number and ρ is the density. Through analytic continuation the equations remain valid in the complex domain. The speed q and angle θ of the flow are related to ξ and η by the formulas

$$\log f(\xi) = \int \sqrt{1-M^2}\,\frac{dq}{q} - i\theta \quad ,$$

$$\log \overline{f(\bar{\eta})} = \int \sqrt{1-M^2}\,\frac{dq}{q} + i\theta \quad ,$$

where f is an analytic function mapping the flow onto a region that can be taken as the unit circle $|\xi| < 1$. Analytic continuation around the sonic line can be performed along paths on which $1-M^2$ does not vanish.

The problem of finding an airfoil on which the pressure is a prescribed function of the arc length is equivalent to the problem of finding a profile on which the speed is assigned as a function $q = q(\phi)$ of the velocity potential. In the unit circle $|\xi| < 1$ this reduces to the question of determining the map function f and the stream function ψ from boundary conditions of the form

$$\mathrm{Re}\{\log f(\xi)\} = h(q) \quad ,$$

$$\mathrm{Re}\{\psi(\xi,\bar{\xi}) - ik\,\phi(\xi,\bar{\xi})\} = 0 \quad ,$$

where h is known in terms of q and k is a given real constant. This nonlinear boundary value problem can be solved iteratively by first guessing ϕ so that f can be calculated and then computing ψ and ϕ so that the process can be repeated. For an appropriate choice of h and k the iterations converge even in the transonic case to a shockless solution that yields the prescribed pressure distribution, except for minor deviations that must be expected in the supersonic zone. (See figures 4 and 5.)

Numerical computations suggest that the boundary value problem that has been formulated in the complex domain is well posed. This has been proved in a very special case by Sanz (ref. 4). He considers the Euler-Poisson-Darboux equation

$$u_{xx} + u_{yy} + \frac{1}{3y}\,u_y = 0$$

obtained by bringing the Tricomi equation into canonical form. The problem becomes to find a solution u in the unit circle with prescribed values of Re{u} on the boundary. Sanz introduces a decomposition

$$u(x,y) = y^{2/3}u_1(x,y^2) + u_2(x,y^2)$$

of u into two solutions that are easily reflected across the x-axis. He is led to boundary value problems for u_1 and u_2 in the upper unit semicircle that can be solved in closed form.

Numerous examples of shockless airfoils have been calculated by the method of complex characteristics using the code with prescribed $q(\phi)$. These include symmetric airfoils with two supersonic zones (figs. 4 and 5), compressor airfoils suitable for stators (fig. 1), and turbine airfoils with large turning angles (fig. 4). The procedure is largely automatic, and it is relatively easy to implement new ideas about design that depend on the pressure distribution. However, further conformal transformation of the characteristic coordinates will be required if gap-to-chord ratios significantly lower than unity are desired.

It would be of interest to generalize the method of designing shockless airfoils based on giving a pressure distribution to the case of three-dimensional transonic flow past a swept wing. This might be achieved by modifying the analysis code appropriately and introducing an artifical viscosity that smears shocks. To start with it would be helpful to have a fast STAR version of the swept wing code published recently by Jameson and Caughey (refs. 2 and 5). Work on these proposals is in progress at the Courant Mathematics and Computing Laboratory.

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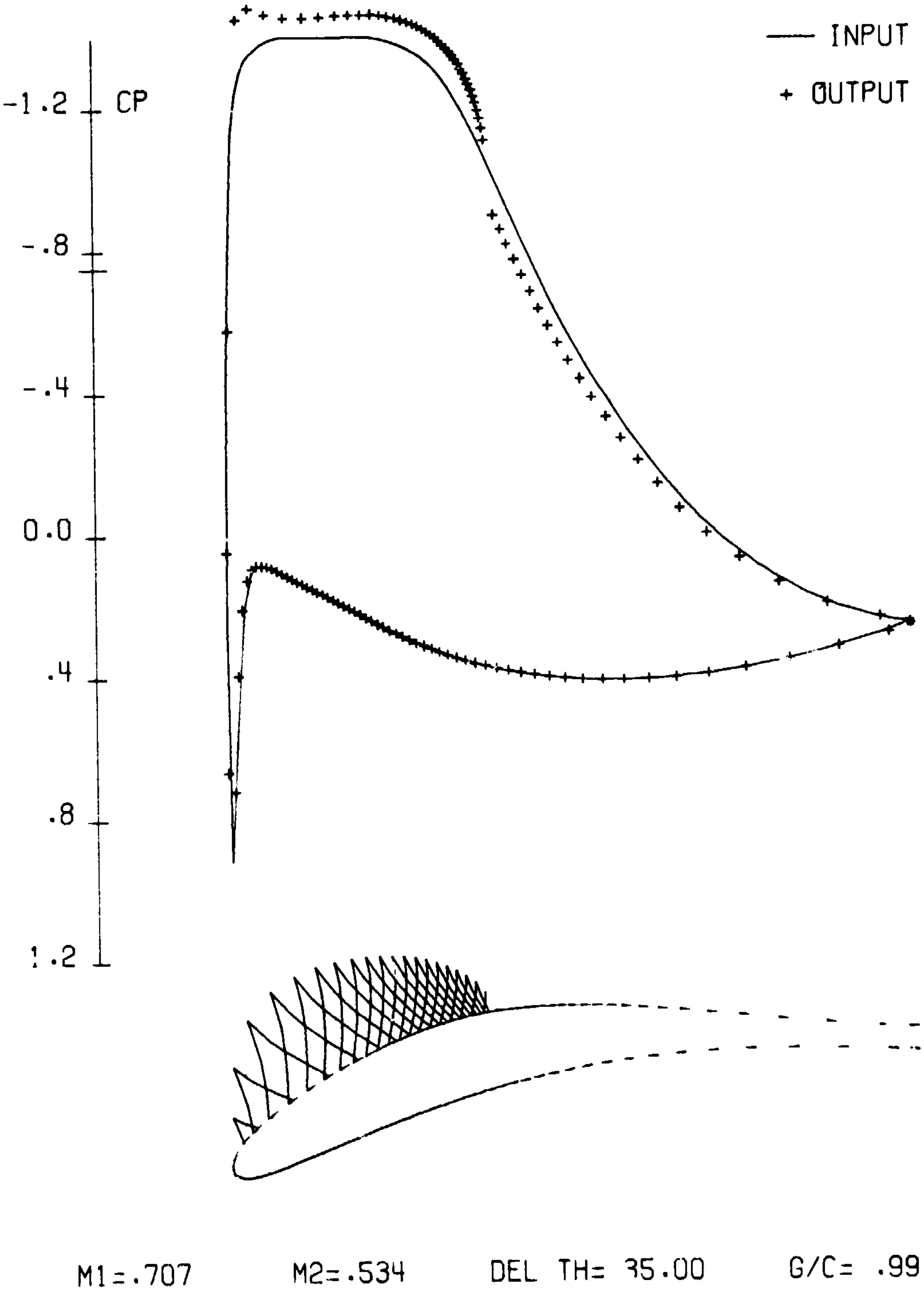
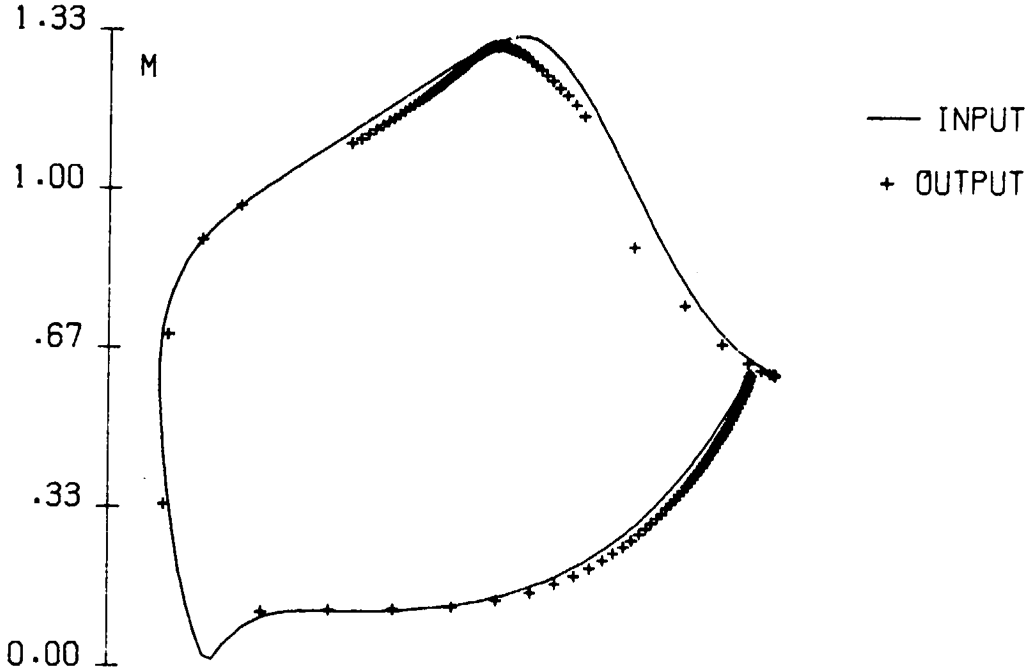
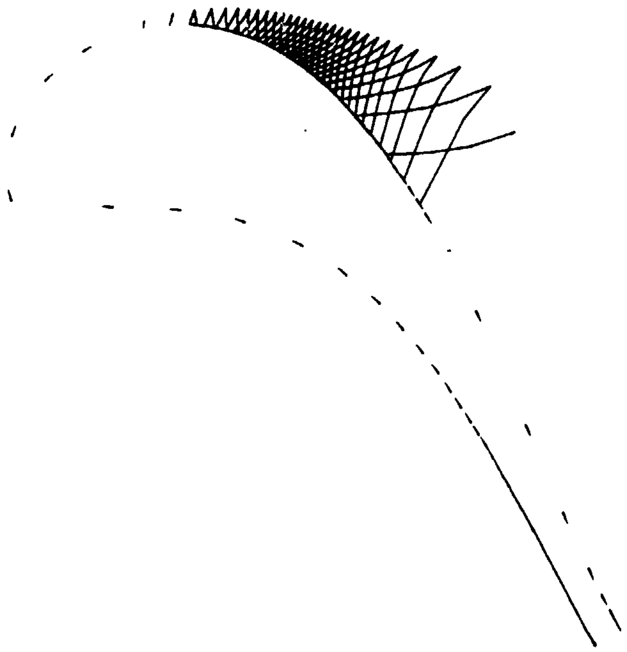


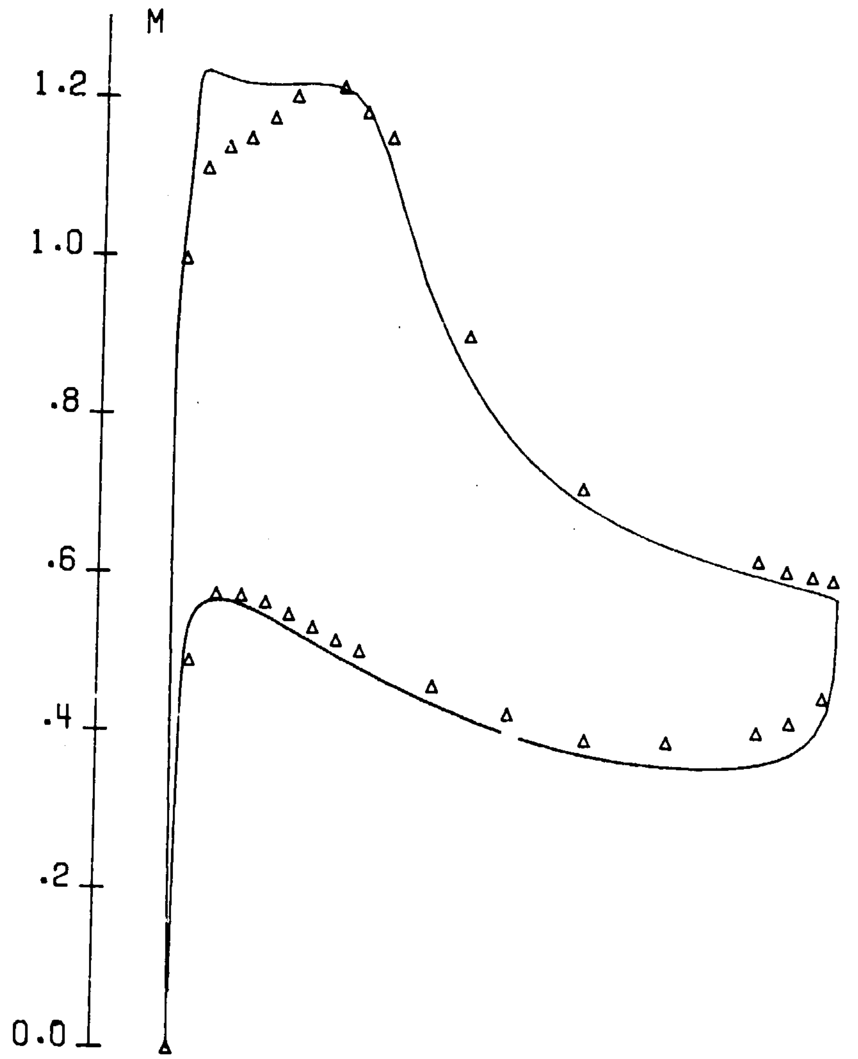
Figure 1.- Stephens stator blade.





M1=.363 M2=.762 DEL TH= 99.26 G/C=1.10

Figure 2.- Sanz turbine blade.

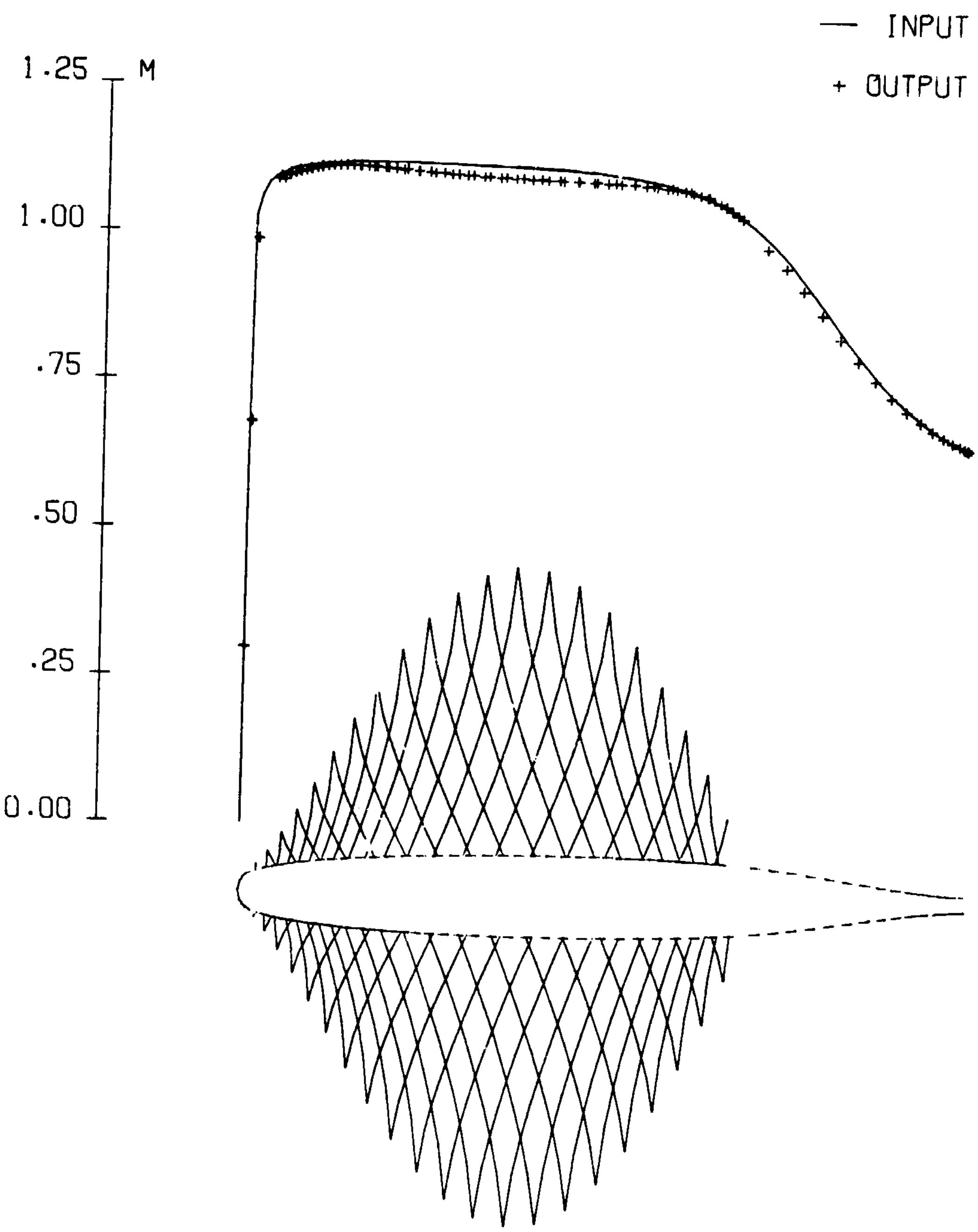


PRATT AND WHITNEY COMPRESSOR AIRFOIL R=1.1 MILLION

— THEORY M1=.780 M2=.480 DEL TH=25.0

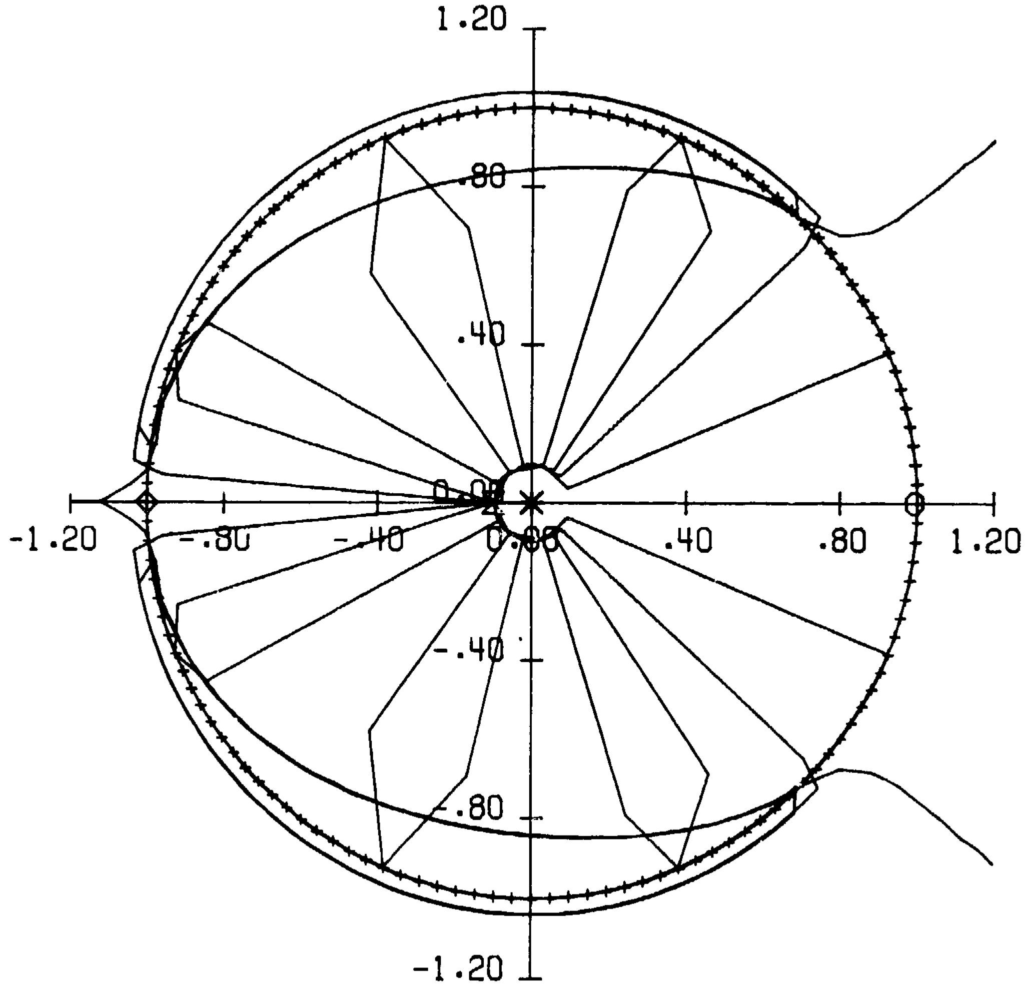
△ EXPERIMENT M1=.775 M2=.544 DEL TH=25.5 LOSS=.0196

Figure 3.- Supercritical compressor test data.



M=.831 CL= .000 DX=-.000 DY= .021 T/C=.110

Figure 4.- Symmetric shockless airfoil.



M=.831 CL= .000 DX=-.000 DY= .021 T/C=.110

Figure 5.- Complex hodograph plane.